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Design Criteria for Optimal Flight Control Systems

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Introduction

THE next generation of aircraft, with increasing stringent demands over a wide range of flight conditions, will necessitate complex augmentation systems requiring multivariable control in order to achieve the desired operational effectiveness in specific tasks, as well as to provide required mission-related flying qualities. The purpose of the reasearch described in this Note is to evolve flight control system design methods that are compatible with current flying qualities specifications and are potentially more systematic and powerful in the sense of achieving flight control objectives quickly and effectively. The multicontroller design method considered is based on linear optimal control theory. Flying qualities requirements can be specified as acceptable regions for poles and zeros of transfer functions of specific response variables with respect to pilot-command inputs. For linear optimal control to be a viable tool, the designer should be able to use it in a systematic fashion for control systems design to satisfy flying qualities. The results presented in this paper demonstrate progress toward this objective. The problem addressed is the selection of performance index matrices and the effects of this selection on the closed-loop poles and transfer function zeros. Two sequential design procedures, one computing the Riccati solution from a set of linear equations and the other computing the closed-loop eigenvectors, are presented that determine at each step, the pole-zero movements of the closed-loop transfer functions as the weighting matrices are varied. A control system design example is also presented.

Characteristics of Linear Optimal Systems

Linear optimal control involves the minimization of a quadratic form of scalar performance index

$$J=\min_u \frac{1}{2}\int_0^\infty (x^TQx+u^TRu)\,dt \tag{1}$$

subject to

$$\dot{x}=Fx+Gu \qquad y=Hx \tag{2}$$

The optimal control law[1] that results is

$$u=-R^{-1}G^TPx=-Kx \tag{3}$$

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where P is the solution of the standard Riccati equation. Substitution of Eq. (3) into Eq. (2) yields the closed-loop transfer functions with respect to the command inputs and hence the closed-loop poles and zeros. Each choice of Q and R corresponds to a different set of closed-loop poles and zeros. The movement of the closed-loop poles can also be determined directly from the root square locus[2] expression

$$\Delta(s)\Delta(-s)=|I+R^{-1}G^T[-sI-F^T]^{-1}Q[sI-F]^{-1}G| \tag{4}$$

as Q and R are varied where $\Delta(s)$ and $\Delta(-s)$ are the closed-loop characteristic polynomials of the optimal system and its adjoint, respectively. Techniques for determining the movement of closed-loop zeros directly, with feedback from multiple inputs, without solving a Riccati equation to compute the gains and transfer functions, are not available except for very simple cases. The response of the closed-loop system is influenced by both the poles and the zeros. For linear optimal control to be a valuable design tool, procedures should be available that determine in a systematic fashion the movement of the closed-loop poles and zeros as the Q and R matrices are varied. In the following section two design procedures are presented that serve as a step toward this objective.

Optimal Control System Design Procedures

The two design procedures presented in this section are an extension of the sequential procedure for pole placement developed by Solheim.[3] This procedure requires the transformation of Eq. (2) to Jordan form. With the transformation $x=Tz$, Eqs. (1) and (2) become

$$\dot{z}=\Lambda z+T^{-1}Gu$$

$$J=\frac{1}{2}\int_0^\infty (z^T\tilde{Q}z+u^TRu)\,dt \qquad \tilde{Q}=T^TQT \qquad Q=T^{-T}\tilde{Q}T^{-1} \tag{5}$$

The procedure determines $\tilde{Q}$, with R fixed, so that only one eigenvalue is shifted at each stage. The matrix Q is computed at each stage from $\tilde{Q}$ and subsequently added together to yield a final Q that moves the open-loop poles to the desired closed-loop locations. This procedure is extended in this paper to include the determination of closed-loop zeros in a sequential manner as Q is varied.

Design Procedure 1

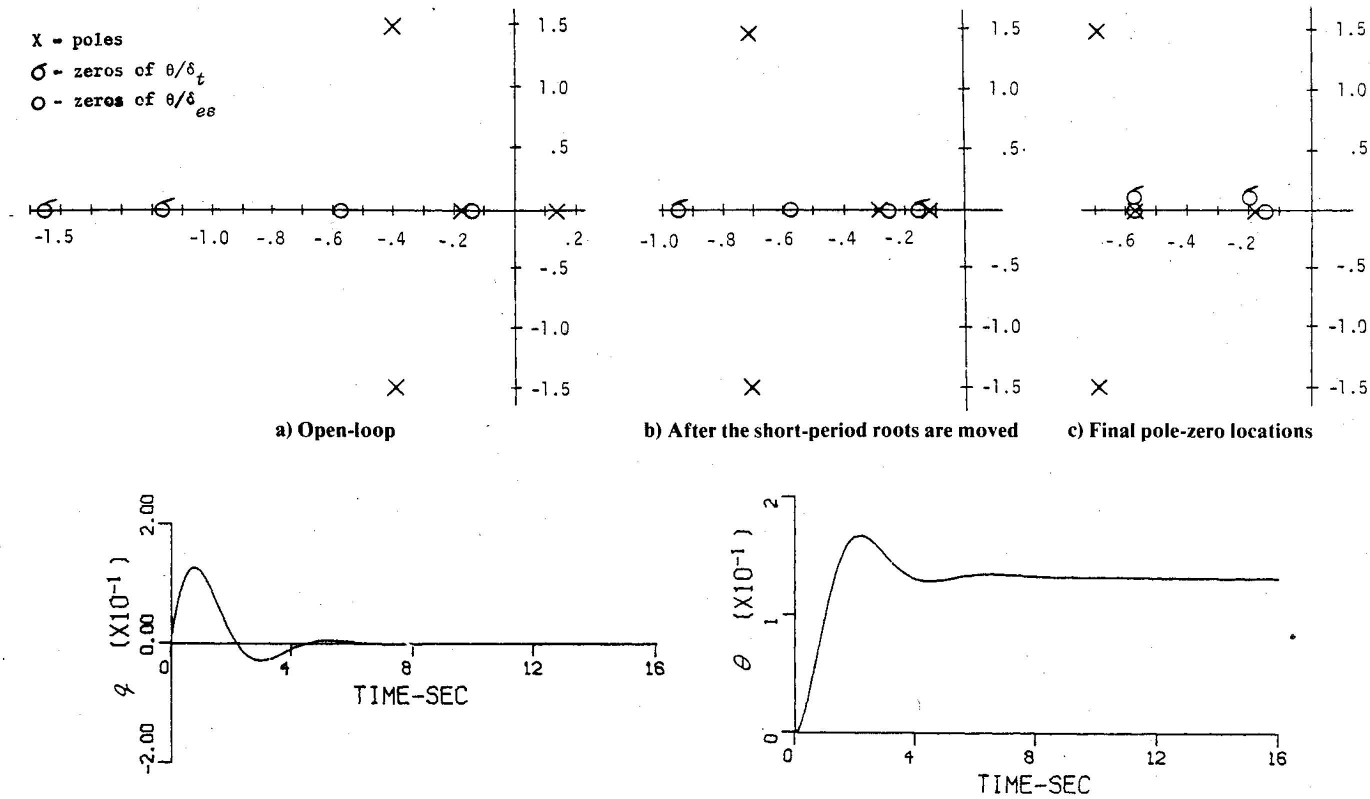
This approach is based upon obtaining the Riccati solution from a set of linear equations.[4] The x transfer functions are obtained from the z transfer functions[5] as

$$x(s)=\frac{T[B_1(s)-B_2(s)\tilde{P}]Gu_c(s)}{\Delta(s)} \tag{6}$$

where $B_1(s)$ and $B_2(s)$ are polynomials in s with elements of $\tilde{Q}$ appearing explicitly, u_c is the comman input, and $\Delta(s)$ is the closed-loop characteristic polynomial. The matrix P is computed as

$$\tilde{P}=B_2^{-1}(s)B_1(s)\,|_{s=s_i} \qquad (i=1,2,...,n) \tag{7}$$

where s is evaluated at s_i, which are mirror images of the closed-loop eigenvalues in the right-half plane. Except for the



a) Open-loop b) After the short-period roots are moved c) Final pole-zero locations

d) Pitch rate and pitch attitude response (deg) to step input

Fig. 1 Pole-zero locations of θ/δ_{es} and θ/δ_t transfer functions and closed-loop response.

eigenvalue that is being shifted, s_i are the same as the open-loop eigenvalues. The movement of closed-loop poles and zeros at each stage is determined by Eq. (6) as $\tilde{Q}$ is varied.

Design Procedure 2

In this design procedure, the eigenvectors of the optimal closed-loop system are used to determine the movement of the closed-loop zeros. The x transfer functions are given as[5]

$$x(s) = TV_{II}\frac{\mathrm{adj}\,[sI-\Lambda_c]}{\Delta(s)}V_{II}^{-1}T^{-1}Gu_c(s) \tag{8}$$

where V_{II} is the matrix of eigenvector for the transformed optimal system z. The matrix V_{II} has a very simple form[5] with the elements of $\tilde{Q}$ appearing explicitly. The inverse of V_{II} can be computed from V_{II} very straightforwardly with the elements of $\tilde{Q}$ appearing explicitly. The matrix Λ_c is diagonal with eigenvalues identical to open-loop values, except for those which are being shifted. The elements of $\tilde{Q}$ appear explicitly in the numerator polynomials of Eq. (8). The numerator polynomials and $\Delta(s)$ determine the movement of closed-loop zeros and poles as $\tilde{Q}$ is varied.

The two design procedures are summarized in the following steps:

1) The system of equations is transformed into Jordan form using the transformation matrix T.

2) The elements of $\tilde{Q}$ are selected, with R fixed, so that only one real pole or a pair of complex poles are moved at each stage. The movement of zeros are determined at each stage from Eq. (6) for design procedure 1 and from Eq. (8) for design procedure 2.

3) The matrix Q and the corresponding gain matrix K are determined.

4) Starting with the new system $F_{\mathrm{new}} = F_{\mathrm{old}} - GK$, the procedure is repeated to determine the movement of other poles and zeros.

In summary, the design procedures described are sequential in determining the matrix Q that moves only one pole and, in general, all the zeros at each stage. The procedures are not explicit zero-placement procedures, but at each stage, the designer has control over their movement.

Illustrative Example

A control system design example is presented in this section using the X-22A V/STOL aircraft as the model. The first procedure presented in the previous section is used to design the control system. The small perturbation, longitudinal, linearized equations of motion can be represented in a general form, as given by Eq. (2), with $x = [\Delta u\ \Delta\omega\ \Delta q\ \Delta\theta]^T$, $u = [\delta_{es}\delta_t]$, $\Delta u = u - u_0$, $\Delta\omega = \omega - \omega_0$, $\Delta q = q - q_0$, and $\Delta\theta = \theta - \theta_0$ where u_0, etc., are reference values, u is the velocity along body x axis (ft/s), ω is the velocity along body z axis (ft/s), q is the body axis pitch rate (deg/s, rad/s), θ is the pitch attitude (deg, rad), δ_{es} is the input which produces pitching moment, and δ_t is the input which produces thrust.

The objective is to design an attitude command augmentation system which has produced satisfactory pilot ratings under instrument flight conditions in the X-22A. The open-loop short-period dynamics are characterized by lightly damped complex roots, while the phugoid has degenerated into a real pair, one of which is unstable as shown in Fig. 1a. The augmentation system employs feedback from both inputs to augment the frequency and damping of the short-period roots and move the zeros of θ transfer functions with respect to the two inputs close to the stabilized aperiodic roots. To achieve the desired objectives, the first design procedure described earlier was used.

In the first step, the two complex poles were moved so that the short-period damping and natural frequency would be in the desired range.[5] This was accomplished by transforming the system to Jordan form and selecting an appropriate matrix $\tilde{Q}_I$ with a particular choice of R. With a different choice of R, a different $\tilde{Q}_l$ would result to achieve the same objectives. The matrix Q_l, the corresponding gain matrix K_l and $F_l = F - GK_l$ were determined.

Thus far the two complex poles have been moved to more desirable locations and the zero movement determined as shown in Fig. 1b. The feedback gains at this stage shifted the open-loop unstable pole to its mirror image in the left half-plane. This is the property of optimal control design. The stable open-loop pole has remained at the same location. The zeros of the transfer function of the pitch attitude to pitch control commands are affected by feedback from the second input, the thrust control command. The elements of the feedback gain matrix corresponding to the feedback from the second input are small and have not moved the zeros of θ/δ_{es} significantly from their open-loop values, but the zeros of θ/δ_t have moved closer to the real poles.

In the next step, one of the real poles is moved close to one of the zeros of θ/δ_{es} and the zeros of θ/δ_t transfer functions moved close to the real poles. To do this, F_l was transformed into block diagonal form and $\tilde{Q}_2$ was selected so that only the real pole closer to the origin and zeros of θ/δ_t were moved close to the zeros of θ/δ_{es} transfer functions as shown in Fig. 1c. The matrices Q_2 and K_2 were computed from $\tilde{Q}_2$. The final closed-loop matrix was computed by $F_c = F_l - GK_2$, and the final performance index matrix $Q = Q_l + Q_2$. It is noted that with sufficiently high feedback gains, one of the real poles and the zeros of θ/δ_t transfer function are driven into close proximity to the zeros of the θ/δ_{es} transfer function, so that the aperiodic pair is close to the zeros of θ/δ_{es} and θ/δ_t transfer functions, and the attitude transfer functions are essentially second order.

The numerator zeros of θ/δ_{es} have moved very little from their open-loop locations. Figure 1c shows the final pole-zero locations and the two real poles approximately cancel the zeros of the θ/δ_{es} and θ/δ_t transfer functions. Thus, the attitude response is essentially second-order dominated by the modified short-period mode. The responses of pitch attitude and pitch rate to a step input are shown in Fig. 1d. The time history illustrates the second-order nature of attitude response. The design example presented in this section has demonstrated the usefulness of the step-by-step approach to control system design. The sequential procedure provides the designer tractable information to select the performance index matrices.

Conclusions

Two sequential design procedures were developed for optimal control system design. These procedures determine the pole-zero movements at each stage as the weighting matrix of the performance index is varied. The weighting matrices constructed at each stage were added to get the final weighting matrix to move the open-loop poles and zeros to more desirable locations. The second design procedure based on the eigenvector approach is considered to be more promising because of increased tractability of the modified system at every stage of the design process, and is being further developed into a systematic design procedure by the authors.

Acknowledgment

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Optimal Member Damper Controller Design for Large Space Structures

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Introduction

CONTROL systems design for large flexible space structures is a complex and challenging problem because of their special dynamic characteristics. Large space structures tend to have extremely low-frequency, lightly damped bending modes which are closely spaced in the frequency domain. Because of pointing requirements, a number of lower frequency modes will probably fall within the bandwidth of the primary controller, thus making active control of some of the modes unavoidable. Since control of all the modes is impractical, the primary controller will be reduced-order. This introduces stability problems because of observation and control spillover.[1-3] The stability of a system with a reduced-order controller is heavily dependent on the natural damping of the residual (uncontrolled) modes. Therefore, it is desirable to increase the damping of the residual modes where possible.

The member damper approach and the application of multiple-member dampers in an output velocity feedback configuration was discussed in Ref. 4. The member damper approach includes local damping elements which could consist of colocated actuators and velocity sensors. Each actuator sensor pair is configured as a single-loop control system and the member dampers work independently of each other. In the output velocity feedback configuration, all the sensor signals are distributed by a gain matrix to interconnect all the actuators and sensors. This concept was further investigated in Refs. 5 and 6. It has been proved in these references that direct velocity feedback (DVFB) with colocated sensors and actuators cannot destabilize the system. Such controllers may be used in conjunction with a conventional (modern) active controller, and have the potential to effect significant improvement in the overall performance.

Selection of velocity feedback gains for individual member dampers is an important part of the design. The root locus technique may be used for this purpose; however, this could be a complex task, especially if a large number of actuators are used. In this note, the problem of selecting velocity feedback gains is formulated as an optimal output feedback regulator problem, and the necessary conditions are derived for minimizing a quadratic performance function. The special structure of the gain matrix (i.e., diagonal) is taken into account, and the knowledge of process noise and sensor noise is used to advantage.

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